

Stability and symmetry breaking in a three-Higgs-doublet model with lepton family symmetry $O(2) \otimes \mathbb{Z}_2$

M. Maniatis,^{1,*} D. Mehta,^{2,3,†} and Carlos M. Reyes^{1,‡}

¹*Departamento de Ciencias Básicas, Universidad del Bío Bío, Casilla 447, Chillán, Chile.*

²*Department of Applied and Computational Mathematics and Statistics,
University of Notre Dame, Notre Dame, IN 46556, USA*

³*Centre for the Subatomic Structure of Matter, Department of Physics,
School of Physical Sciences, University of Adelaide, Adelaide, South Australia 5005, Australia.*

Motivated by the neutrino data, an extension of the Standard Model with three Higgs-boson doublets has been proposed. Imposing an $O(2) \otimes \mathbb{Z}_2$ family symmetry, a neutrino mixing matrix with $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ appears in a natural way. Even though these values for the mixing matrix do not follow the recent experimental constraints, they are nevertheless a good approximation. We study the Higgs potential of this model in detail. We apply recent methods which allow for the study of any three-Higgs-boson doublet model. It turns out that for a variety of parameters the potential is stable, has the correct electroweak symmetry breaking, and gives the correct vacuum expectation value.

1. THE $O(2) \otimes \mathbb{Z}_2$ MODEL

The experimental neutrino mixing data show that the neutrino mixing is very different from the quark mixing. In the usual parametrization of the neutrino mixing matrix (see for instance [1]), experimental data suggest that the angle θ_{13} is small (but nonzero), and θ_{23} is close to $\pi/4$ [2].

There is lot of effort spent on finding an organizing principle for the flavor puzzle. A general approach is to study finite subgroups of $SU(3)$ which have an irreducible triplet representation; see for instance [3] and references therein. Examples of subgroups with triplet representations are S_4 , the group of permutations of 4 elements, with two singlet, one doublet, and two triplet representations (see for instance [4]); or A_4 , the group of even permutations of 4 elements which has also a triplet representation besides singlet representations (see for instance [5, 6]); or $\Delta(27)$, with 27 elements and two triplet representations (see for instance [7]).

In contrast, here we want to study in detail the Higgs potential of a three Higgs-boson-doublet model which imposes a $O(2) \times \mathbb{Z}_2$ symmetry – without any irreducible triplet representation. Let us closely follow the motivation of [8]. The starting point is a neutrino mass matrix which is symmetric in generations two and three,

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}. \quad (1.1)$$

This mass matrix may be diagonalized as usual, that is, $U^\dagger M_\nu U = \text{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2)$, where U is the neutrino mixing matrix and $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ the neutrino masses.

Expressing the mixing matrix U in terms of the usual parametrization [1], we get in particular $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Even though the experimental results are not in exact agreement with these values, in particular θ_{13} is nonzero, they at least appear to be approximately fulfilled. We note that deviations from these approximate values for the mixing angles could arise by soft-symmetry-breaking terms or beyond-leading-order effects; see for instance the study in [9].

The mass matrix (1.1) may be generated by the introduction of three Higgs-boson doublets φ_i , $i = 1, 2, 3$, and a symmetry $O(2) \times \mathbb{Z}_2 \cong \mathbb{Z}'_2 \times \mathbb{Z}_2 \times U(1)$, where all elementary particles are assigned to an appropriate transformation behavior on this symmetry. The reflection symmetry \mathbb{Z}'_2 is responsible for the μ - τ symmetry of (1.1):

$$\mathbb{Z}'_2 : D_{\mu L} \leftrightarrow D_{\tau L}, \mu_R \leftrightarrow \tau_R, \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \phi_1 \leftrightarrow \phi_2 \quad (1.2)$$

Here $D_{\mu L}$ and $D_{\tau L}$ denote the left-handed $SU(2)$ lepton doublets, $\nu_{\mu R}$ and $\nu_{\tau R}$ the right-handed neutrinos, and all remaining fields transform trivially under the \mathbb{Z}'_2 symmetry. The \mathbb{Z}_2 symmetry is given by a sign change,

$$\mathbb{Z}_2 : \nu_{eR} \rightarrow -\nu_{eR}, \nu_{\mu R} \rightarrow -\nu_{\mu R}, \nu_{\tau R} \rightarrow -\nu_{\tau R}, \\ e_R \rightarrow -e_R, \varphi_3 \rightarrow -\varphi_3, \quad (1.3)$$

with $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ the right-handed neutrinos, e_R the right-handed electron, and all other fields unchanged under this \mathbb{Z}_2 symmetry. Eventually the assignment with respect to the phase symmetry $U(1)$ is

$$U(1) : \begin{array}{c|ccc} X \longrightarrow e^{i\theta} X & D_{\mu L}, \tau_R, \nu_{\mu R} & D_{\tau L}, \mu_R, \nu_{\tau R} & \varphi_1 & \varphi_2 \\ \hline \theta & 1 & -1 & 2 & -2 \end{array} \quad (1.4)$$

with X one of the fields on the right-hand side of the table transforming as $X \longrightarrow e^{i\theta} X$ with the corresponding

*E-mail: MManiatis@ubiobio.cl

†E-mail: dmehta@nd.edu

‡E-mail: creyes@ubiobio.cl

phase θ given explicitly in the table. All other fields transform trivially.

By virtue of these symmetries – besides the electroweak $SU(2)_L \times U(1)_Y$ symmetry – there appear in particular the invariant Yukawa couplings

$$\mathcal{L}_Y = -y_4 (\bar{D}_{\mu L} \varphi_1 \mu_R + \bar{D}_{\tau L} \varphi_2 \tau_R) + h.c. \quad (1.5)$$

The most general potential for the three Higgs-boson doublets $\varphi_1, \varphi_2, \varphi_3$ reads

$$\begin{aligned} V_{O(2) \times \mathbb{Z}_2} = & \mu_0 \varphi_3^\dagger \varphi_3 + \mu_{12} \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) \\ & + \mu_m \left(\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1 \right) + a_1 (\varphi_3^\dagger \varphi_3)^2 \\ & + a_2 \varphi_3^\dagger \varphi_3 \left(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \right) \\ & + a_3 \left(\varphi_3^\dagger \varphi_1 \cdot \varphi_1^\dagger \varphi_3 + \varphi_3^\dagger \varphi_2 \cdot \varphi_2^\dagger \varphi_3 \right) \\ & + a_4 \varphi_3^\dagger \varphi_1 \cdot \varphi_3^\dagger \varphi_2 + a_4^* \varphi_1^\dagger \varphi_3 \cdot \varphi_2^\dagger \varphi_3 \\ & + a_5 \left((\varphi_1^\dagger \varphi_1)^2 + (\varphi_2^\dagger \varphi_2)^2 \right) \\ & + a_6 \varphi_1^\dagger \varphi_1 \cdot \varphi_2^\dagger \varphi_2 + a_7 \varphi_1^\dagger \varphi_2 \cdot \varphi_2^\dagger \varphi_1, \quad (1.6) \end{aligned}$$

where the term $\mu_m (\varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1)$ breaks the $U(1)$ symmetry (1.4) explicitly but softly, since this is a quadratic term. For a nonvanishing parameter μ_m in this way additional Goldstone bosons are avoided, which otherwise would appear by spontaneous symmetry breaking of the $U(1)$ symmetry. This potential has nine real parameters and one complex parameter a_4 , corresponding to eleven real parameters in total.

Now we want to discuss stability, stationarity, and electroweak symmetry breaking of this model. Of course only a model with a stable potential, having the correct electroweak-symmetry-breaking behavior and the correct vacuum expectation values is physically acceptable. These obvious constraints restrict the parameter space of the potential. Here, we focus on the Higgs potential and not on any further experimental limits. For instance, the expressions for the *oblique parameters* S, T, U are available for any nHDM [10] and may be compared to the electroweak precision data.

Since the $O(2) \otimes \mathbb{Z}_2$ model is a 3HDM we encounter in this model four charged Higgs bosons and in total five neutral Higgs bosons. We expect a changed phenomenology of this model compared to the Standard Model. Of course, the detection of any further Higgs boson would be a clear signal for a model beyond the Standard Model.

Depending on the Yukawa coupling strength in (1.5) we have for instance the signature of the production of a charged Higgs boson with subsequent decay into a muon and a muon-neutrino. The potential itself is in principle detectable via its trilinear and quartic Higgs self-couplings. We leave this investigation for future work and focus here on the study of the Higgs potential with respect to stability, electroweak symmetry breaking and the global minimum.

Even though the potential (1.6) appears to be rather involved we will see that it is indeed accessible in the bilinear approach [11–13]. In this approach gauge degrees of freedom are avoided systematically. Moreover, the corresponding equations for stability and stationarity simplify in particular, the degree of systems of equations is lowered. Recently, the bilinear approach for the study of stability, stationarity, and electroweak symmetry breaking has been extended to the study of any 3HDM [14], which we now briefly review.

The scalar products of the type $\varphi_i^\dagger \varphi_j$, $i, j \in \{1, 2, 3\}$, in the potential (1.6) may be arranged in a 3×3 matrix

$$\underline{K} = \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \varphi_2^\dagger \varphi_1 & \varphi_3^\dagger \varphi_1 \\ \varphi_1^\dagger \varphi_2 & \varphi_2^\dagger \varphi_2 & \varphi_3^\dagger \varphi_2 \\ \varphi_1^\dagger \varphi_3 & \varphi_2^\dagger \varphi_3 & \varphi_3^\dagger \varphi_3 \end{pmatrix}. \quad (1.7)$$

By the introduction of the bilinears,

$$K_\alpha = K_\alpha^* = \text{tr}(\underline{K} \lambda_\alpha), \quad \alpha = 0, \dots, 8 \quad (1.8)$$

with λ_α the 3×3 Gell-Mann matrices, the following replacements can be made in the potential:

$$\begin{aligned} \varphi_1^\dagger \varphi_1 &= \frac{K_0}{\sqrt{6}} + \frac{K_3}{2} + \frac{K_8}{2\sqrt{3}}, & \varphi_1^\dagger \varphi_2 &= \frac{1}{2} (K_1 + iK_2), \\ \varphi_1^\dagger \varphi_3 &= \frac{1}{2} (K_4 + iK_5), & \varphi_2^\dagger \varphi_2 &= \frac{K_0}{\sqrt{6}} - \frac{K_3}{2} + \frac{K_8}{2\sqrt{3}}, \\ \varphi_2^\dagger \varphi_3 &= \frac{1}{2} (K_6 + iK_7), & \varphi_3^\dagger \varphi_3 &= \frac{K_0}{\sqrt{6}} - \frac{K_8}{\sqrt{3}}. \end{aligned} \quad (1.9)$$

Comparing the potential, written in terms of bilinears

$$V = \xi_\alpha K_\alpha + K_\alpha \tilde{E}_{\alpha\beta} K_\beta, \quad \alpha, \beta = (0, \dots, 8), \quad (1.10)$$

with the general form of the potential, we find the new parameters

$$\begin{aligned}
(\xi_\alpha) &= \left(\frac{\mu_0 + 2\mu_{12}}{\sqrt{6}}, \mu_m, 0, 0, 0, 0, 0, \frac{\mu_{12} - \mu_0}{\sqrt{3}} \right)^T, \\
(\tilde{E}_{\alpha\beta}) &= \frac{1}{4} \cdot \begin{pmatrix} \frac{2}{3}(a_1 + 2a_2 + 2a_5 + a_6) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{8}}{3}(-a_1 - \frac{a_2}{2} + a_5 + \frac{a_6}{2}) \\ 0 & a_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2a_5 - a_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_3 & 0 & \text{Re}a_4 & \text{Im}a_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_3 & \text{Im}a_4 & -\text{Re}a_4 & 0 \\ 0 & 0 & 0 & 0 & \text{Re}a_4 & \text{Im}a_4 & a_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{Im}a_4 & -\text{Re}a_4 & 0 & a_3 & 0 \\ \frac{\sqrt{8}}{3}(-a_1 - \frac{a_2}{2} + a_5 + \frac{a_6}{2}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4}{3}a_1 - \frac{4}{3}a_2 + \frac{2}{3}a_5 + \frac{1}{3}a_6 \end{pmatrix}.
\end{aligned} \tag{1.11}$$

Obviously, all parameters are real in terms of bilinears. We note that there is a one-to-one correspondence between the Higgs-boson doublets and the bilinear matrix $\underline{K} = 1/2 K_\alpha \lambda_\alpha$ with rank smaller or equal to two - except for irrelevant gauge degrees of freedom; see [12].

Supposing the potential is bounded from below, at the global minimum, or the degenerate minima, the gradient of the potential has to vanish. The corresponding equations may be used to fix some of the parameters. In order to obtain these equations we start with the parametrization of the three Higgs-boson doublets with the same hypercharge $y = +1/2$, in a particular gauge,

$$\begin{aligned}
\varphi_{1/2}(x) &= \begin{pmatrix} \phi_{1/2}^+(x) \\ \frac{1}{\sqrt{2}}(v_{1/2} + H_{1/2}^0(x) + iA_{1/2}^0(x)) \end{pmatrix}, \\
\varphi_3(x) &= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v_3 + h^0(x)) \end{pmatrix}.
\end{aligned} \tag{1.12}$$

The derivatives of the potential (1.6), inserting (1.12) with respect to the fields at the vacuum, that is, for vanishing fields give the nontrivial conditions

$$\begin{aligned}
\mu_0 &= -\frac{1}{2}(a_2 + a_3)(v_1^2 + v_2^2) - a_1 v_3^2 - \text{Re}(a_4)v_1 v_2, \\
\mu_{12} &= -\frac{1}{2}(a_2 + a_3)v_3^2 - a_5(v_1^2 + v_2^2), \\
\mu_m &= \frac{1}{2}((2a_5 - a_6 - a_7)v_1 v_2 - \text{Re}(a_4)v_3^2), \\
\text{Im}(a_4) \cdot v_2 v_3^2 &= 0, \\
\text{Im}(a_4) \cdot v_1 v_3^2 &= 0.
\end{aligned} \tag{1.13}$$

For nonvanishing vacuum expectation values, the last two equations immediately dictate that a_4 has to be real. Eventually, by means of the equations (1.13) the quadratic parameters μ_0 , μ_{12} , μ_m may be expressed by the quartic parameters and the three vacuum expectation values v_1 , v_2 , v_3 . Further, the vacuum expectation values

are restricted with view on the Yukawa couplings (1.5), that is, the ratio of the vacuum expectation values v_1 and v_2 has to be $v_1/v_2 = m_\mu/m_\tau$ at tree-level accuracy. In addition, the vacuum expectation value

$$v_0 = \sqrt{v_1^2 + v_2^2 + v_3^2} \approx 246 \text{ GeV} \tag{1.14}$$

is given by the electroweak precision data. Therefore, all quadratic parameters follow from the quartic parameters and one free vacuum expectation value, say v_3 . Therefore, it appears reasonable to start with the following set of parameters,

$$\begin{aligned}
&a_1, a_2, a_3, \text{Re}(a_4), a_5, a_6, a_7, v_3, \\
&v_0 \approx 246 \text{ GeV}, v_1/v_2 = m_\mu/m_\tau.
\end{aligned} \tag{1.15}$$

Note that the tadpole conditions (1.13) only ensure that there is at least one stationary solution. By no means does this guarantee that the corresponding potential is stable and has a global minimum with the correct partially broken electroweak symmetry.

2. STABILITY AND ELECTROWEAK SYMMETRY BREAKING IN THE $O(2) \otimes \mathbb{Z}_2$ MODEL

In this section we analyze the potential of the $O(2) \otimes \mathbb{Z}_2$ model while varying two of its parameters. Starting with the parameters (1.15) we fix the quadratic parameters μ_0 , μ_{12} , μ_m by (1.13). Quantitatively, we choose the quartic parameters motivated by the central point given in [8] with a variation of the two parameters $a_1 \in [0, 5]$ and $a_2 \in [-3, 3]$ in steps of 0.2:

$$\begin{aligned}
&a_1 \in [0, 5], a_2 \in [-3, 3], a_3 = -5, a_4 = -0.0474, \\
&a_5 = 1.5, a_6 = 2, a_7 = 3, \\
&v_1/v_2 = 105.66/1776.82, v_3 = v_0/\sqrt{2},
\end{aligned} \tag{2.1}$$

with the masses of the muon, m_μ , and tau, m_τ , given in [1]. The central point in particular passes the electroweak precision observables – for details, see [8].

First, we study the stability of the potential. Therefore, we separate the potential into the quadratic and quartic parts $V = K_0 J_2 + K_0^2 J_4$, with J_2 and J_4 given by

$$\begin{aligned} J_2(k_a) &= \frac{\mu_0 + 2\mu_{12}}{\sqrt{6}} + \left(\frac{\mu_{12} - \mu_0}{\sqrt{3}} \right) k_8 + \mu_m k_1, \\ J_4(k_a) &= \frac{1}{6}(a_1 + 2a_2 + 2a_5 + a_6) \\ &\quad + \frac{1}{3\sqrt{2}}(-2a_1 - a_2 + 2a_5 + a_6)k_8 + \frac{a_7}{4}(k_1^2 + k_2^2) \\ &\quad + \frac{1}{4}(2a_5 - a_6)k_3^2 + \frac{a_3}{4}(k_4^2 + k_5^2 + k_6^2 + k_7^2) \\ &\quad + \frac{\text{Re}(a_4)}{2}(k_4 k_6 - k_5 k_7) + \frac{\text{Im}(a_4)}{2}(k_4 k_7 + k_5 k_6) \\ &\quad + \frac{1}{12}(4a_1 - 4a_2 + 2a_5 + a_6)k_8^2, \end{aligned} \quad (2.2)$$

with the vector components $k_a = K_a/K_0$, $a = 1, \dots, 8$ defined for $K_0 \neq 0$. For $K_0 = 0$ the potential vanishes. The stationary points of $J_4(k_a)$ corresponding to a matrix \underline{K} with rank 2 are obtained from

$$\begin{aligned} \nabla_{k_1, \dots, k_8} \left[J_4(k_a) - u \left(\det(\sqrt{2/3} \mathbb{1}_3 + k_a \lambda_a) \right) \right] &= 0, \\ \det(\sqrt{2/3} \mathbb{1}_3 + k_a \lambda_a) &= 0, \\ 2 - k_a k_a &> 0, \end{aligned} \quad (2.3)$$

and the stationary points corresponding to a matrix \underline{K} with rank 1 are obtained from

$$\begin{aligned} \nabla_{\mathbf{w}^\dagger} \left[J_4(k_a(\mathbf{w}^\dagger, \mathbf{w})) - u (\mathbf{w}^\dagger \mathbf{w} - 1) \right] &= 0, \\ \mathbf{w}^\dagger \mathbf{w} - 1 &= 0 \end{aligned} \quad (2.4)$$

where we express the vector components k_a by

$$k_a(\mathbf{w}^\dagger, \mathbf{w}) = \sqrt{\frac{3}{2}} \mathbf{w}^\dagger \lambda_a \mathbf{w} \quad (2.5)$$

and \mathbf{w} , \mathbf{w}^\dagger are three-component complex vectors. If we have for all real solutions k_a of the systems of polynomial equations (2.3) and (2.4) $J_4(k_a) > 0$, or at least $J_4(k_a) = 0$ but then in addition $J_2(k_a) \geq 0$, the potential is stable. In other words, if there is for a given initial parameter set one solution with $J_4(k_a) < 0$ or $J_4(k_a) = 0$ but in addition $J_2(k_a) < 0$ the potential is unstable. The unstable cases for the variation of parameters (2.1) are denoted by the larger full disks (blue) in Fig. 1. For all other values of parameters the potential is stable.

Let us note that the quartic parameters a_1 and a_2 appear as coefficients of $(\varphi_3^\dagger \varphi_3)^2$ and $\varphi_3^\dagger \varphi_3 (\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2)$,

respectively, in the potential (1.6). Therefore it is evident that the potential is unstable for small values of a_1 and too negative values for a_2 .

Having determined parameter sets giving a stable potential we proceed with the study of the stationary points in these cases. We systematically look for all stationary points of the potential. To this end we have to solve the following systems of polynomial equations, corresponding to solutions which break electroweak symmetry partially (conserving the electromagnetic $U(1)_{em}$ symmetry), and solutions which break the electroweak symmetry completely.

The stationary solutions with full electroweak symmetry breaking, corresponding to stationarity matrices $\underline{K} = K_\alpha \lambda_\alpha / 2$ of rank 2 are obtained from

$$\begin{aligned} \nabla_{K_0, \dots, K_8} \left[V(K_0, \dots, K_8) - u \det(\underline{K}) \right] &= 0, \\ 2K_0^2 - K_a K_a &> 0, \\ \det(\underline{K}) &= 0, \\ K_0 &> 0. \end{aligned} \quad (2.6)$$

The stationary solutions with partial electroweak symmetry breaking, corresponding to stationarity matrices $\underline{K} = K_\alpha \lambda_\alpha / 2$ of rank 1 are obtained from

$$\begin{aligned} \nabla_{\mathbf{w}^\dagger, K_0} \left[V(K_\alpha(K_0, \mathbf{w}^\dagger, \mathbf{w})) - u (\mathbf{w}^\dagger \mathbf{w} - 1) \right] &= 0, \\ \mathbf{w}^\dagger \mathbf{w} - 1 &= 0, \\ K_0 &> 0 \end{aligned} \quad (2.7)$$

where we express the bilinears K_α in terms of K_0 and the three component complex vectors \mathbf{w} and \mathbf{w}^\dagger ,

$$K_\alpha(K_0, \mathbf{w}^\dagger, \mathbf{w}) = \sqrt{\frac{3}{2}} K_0 \mathbf{w}^\dagger \lambda_\alpha \mathbf{w}, \quad \alpha = 0, \dots, 8. \quad (2.8)$$

In (2.6) and (2.7), u is a Lagrange multiplier, respectively.

In addition there is always a solution for a vanishing potential, corresponding to an unbroken electroweak symmetry.

The global minimum, that is, the vacuum, is given by the stationary point or points with the deepest potential value. In case this solution originates from the set (2.7), that is, when we do have electroweak symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, we can directly calculate the vacuum-expectation value of this minimum, $v_0^2 = \sqrt{6} K_0$, and verify that it coincides with (1.14).

Depending on the variation of the parameters a_1 and a_2 we detect the viable global minima. These cases are marked by little (green) dots in Fig. 1. In case the deepest potential value does not correspond to the correct electroweak symmetry breaking or does not give the correct vacuum expectation value, these parameter points are denoted by a circle (red) in Fig. 1.

As we can see by the scattering of points, we typically find valid parameters for a_1 and a_2 not too small. However, the pattern of valid points appears very sensitive to

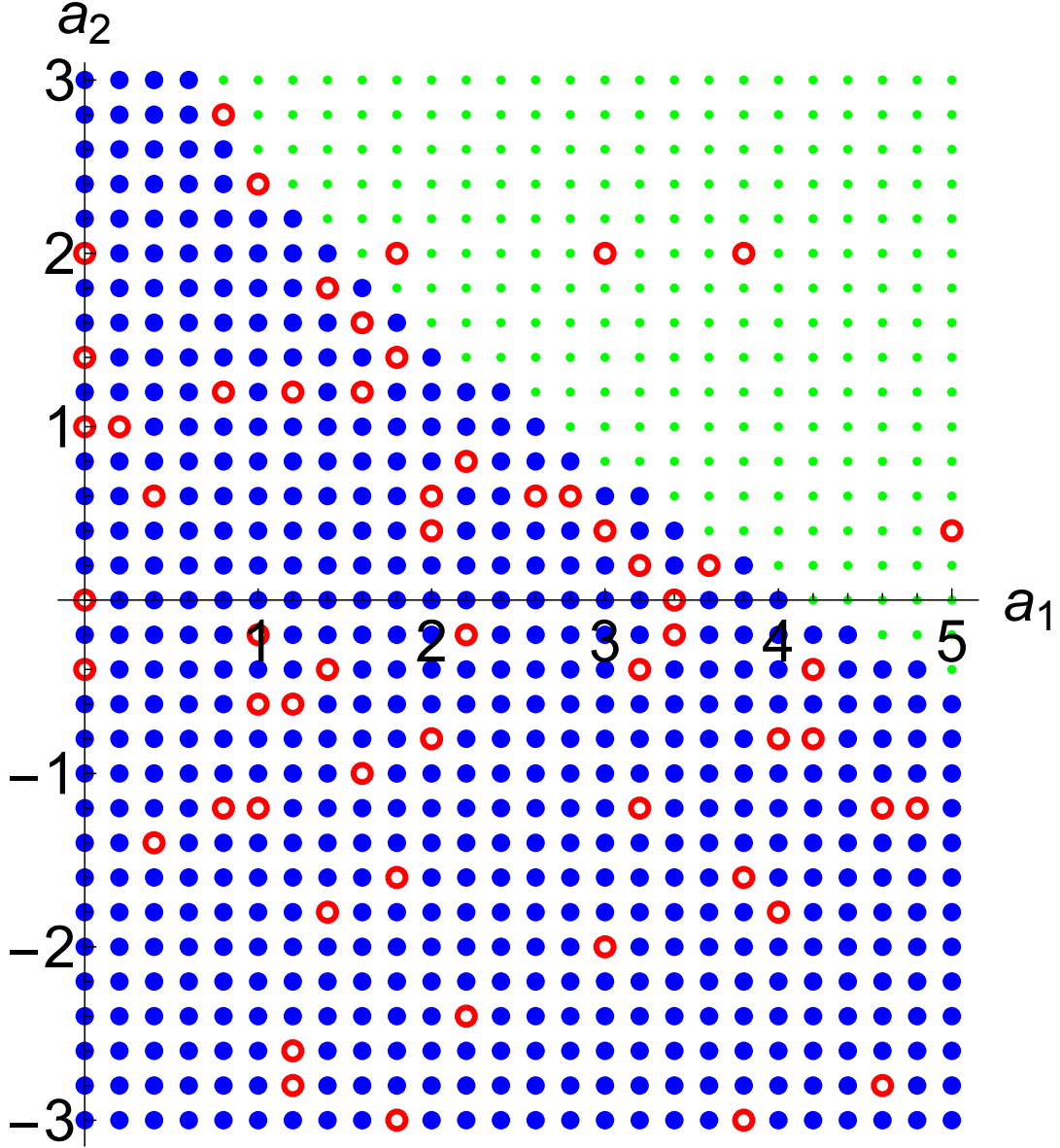


FIG. 1: Stability and stationarity solutions of the 3HDM Higgs potential, varying the two quartic parameters a_1 and a_2 of the potential (1.6). The other quartic parameters are set to $a_3 = -5$, $a_4 = -0.0474$, $a_5 = 1.5$, $a_6 = 2$, $a_7 = 3$. The quadratic parameters are fixed by the equations (1.13), the vacuum expectation value v_0 , and the ratio $v_1/v_2 = m_\mu/m_\tau$ as well as $v_3 = v_0/\sqrt{2}$. The larger full circles (blue) show points where the potential is unstable. The open circles (red) show parameters where no correct electroweak symmetry breaking appears or the vacuum expectation value v_0 is unequal to 246 GeV. Finally, the small dots (green) have a viable global minimum corresponding to the correct vacuum expectation value.

the parameter values. This, of course, is a consequence of the rather involved potential (1.6).

Eventually, let us remark on the technical aspects to solve the rather involved systems of equations - on the one hand for the study of stability (2.3), (2.4), and on the other hand for the study of stationarity (2.6), (2.7). We apply for all the polynomial systems of equations the homotopy continuation approach as implemented in the PHCpack package [15]. For a brief introduction to the homotopy continuation method, see for instance [16].

In the case of the systems of equations (2.4), (2.7), we decompose the three component complex vectors \mathbf{w}^\dagger , \mathbf{w} into real and imaginary parts. In turn we split every equation in the sets into real and imaginary parts. In this way, all indeterminants in all systems of equations have to be real, and we discard all nonreal solutions. Practically, we treat any solution as real if the imaginary part of any of the indeterminants has an amount smaller than 0.001.

Let us study some details of the potential in case it has the correct electroweak symmetry breaking

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. This is conveniently done in a new basis

$$\varphi'_i(x) = U_{ij}\varphi_j(x), \quad i, j \in \{1, 2, 3\} \quad (2.9)$$

in which only the field $\varphi'_3(x)$ gets a nonvanishing vacuum expectation value $v_0/\sqrt{2}$. The unitary matrix U is determined by two rotation angles, which have to fulfill

$$\begin{aligned} v_1 &= \sin(\beta_2) \sin(\beta_1) v_0, \\ v_2 &= \sin(\beta_2) \cos(\beta_1) v_0, \\ v_3 &= \cos(\beta_2) v_0 \end{aligned} \quad (2.10)$$

with $\beta_1 \in [0, 2\pi[$, $\beta_2 \in [0, \pi]$ and we have in particular $v_1^2 + v_2^2 + v_3^2 = v_0^2$. In the new basis, the bilinear parameter vector (ξ_α) from (1.11) becomes

$$(\xi'_\alpha) = \begin{pmatrix} \xi_0 \\ \cos(2\beta_1) \cos(\beta_2) \xi_1 \\ 0 \\ \sin(\beta_1) \cos(\beta_1) (1 + \cos^2(\beta_2)) \xi_1 + \sqrt{3}/2 \sin^2(\beta_2) \xi_8 \\ -\cos(2\beta_1) \sin(\beta_2) \xi_1 \\ 0 \\ \sin(\beta_2) \cos(\beta_2) (\sin(2\beta_1) \xi_1 - \sqrt{3} \xi_8) \\ 0 \\ \sqrt{3} \cos(\beta_1) \sin(\beta_1) \sin^2(\beta_2) \xi_1 + \frac{1}{4} (1 + 3 \cos(2\beta_2)) \xi_8 \end{pmatrix} \quad (2.11)$$

The correct global minimum has, in terms of these parameter components, the potential value

$$\langle V \rangle = \frac{v_0^2}{2\sqrt{6}} (\xi'_0 - \sqrt{2} \xi'_8). \quad (2.12)$$

This result serves as a good cross-check of the numerical study. In particular, the potential value at vacuum has to be nonpositive in order not to lie above the unbroken minimum, which is given by a vanishing potential.

The physical Higgs bosons follow from the diagonalization of the charged and neutral squared Higgs mass matrices. The neutral field of the third Higgs boson φ_3 , that is, $h^0(x)$, is a mass eigenstate if the parameters fulfill

$$\xi'_4 = \xi'_5 = \xi'_6 = \xi'_7 = 0. \quad (2.13)$$

Therefore with a view on (2.11), we find the explicit conditions for the $O(2) \otimes \mathbb{Z}_2$ model, where the neutral component $h^0(x)$ is *aligned* with the vacuum expectation value. The mass squared of the neutral component is in this case

$$m_{h^0}^2 = \frac{4}{\sqrt{3}} (\xi'_8 - \frac{1}{\sqrt{2}} \xi'_0). \quad (2.14)$$

Alignment thus requires that either $v_1 = v_2$ or $\mu_m = 0$ which is not the case, since the ratio of the two vacuum expectation values v_1 and v_2 is fixed by the ratio of the muon and tau masses, and a nonvanishing parameter μ_m is required in order to break the $O(2) \otimes \mathbb{Z}_2$ symmetry softly.

Let us comment on the neutrino mixing angles $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Since these values seem not to be exactly fulfilled experimentally, we mention that deviations may be achieved by imposing further soft-breaking terms in the potential (1.6). We have seen that the value and the nature of the global minimum is rather sensitive to small changes of the potential. Therefore, we expect that definite results would require a separate study of the changed potential. However, since additional soft-breaking terms do not affect the quartic part of the potential, and, in particular, we have found that for large parts of parameter space stability follows from the quartic terms alone, we expect stability also in the respective cases of a potential imposing additional soft-breaking terms.

We would like to mention that the parameters a_2 , a_3 , and a_4 couple the neutral boson $h^0(x)$ with the other two generations of doublets. In general this may lead to changed phenomenology in Higgs boson production and decay of the $h^0(x)$ field. For some investigation of this we refer to [8]. A further detailed study for instance, with respect to a possible enhancement of the $h^0(x)$ decay rate into a pair of photons is left for future work.

3. CONCLUSIONS

The $O(2) \otimes \mathbb{Z}_2$ model [8] introduces three Higgs-boson doublets accompanied by an appropriate assignment of the elementary particles to irreducible representations of the group. In this way a neutrino mass matrix is generated which corresponds to mixing angles which are close to the experimental measurements. However, even though the symmetry restricts the model, the Higgs potential appears to be rather involved. Nevertheless, the recently introduced methods to study any three-Higgs doublet model [14] were applied to study the potential in detail. We have investigated stability, the stationary points, and electroweak symmetry breaking of the Higgs potential by solving the corresponding stationary equations, employing polynomial homotopy continuation. The method numerically finds all the isolated complex solutions out of which we have extracted the physical real solutions. We have scanned over a range of values of the potential parameters. As expected, for too low values of the quartic parameters typically an unstable potential is encountered. For parameter values, corresponding to a stable potential, the global minimum was detected. Our study reveals that in this model there are viable parameters corresponding to a stable global minimum with correct electroweak symmetry breaking.

Acknowledgement

We would like to thank Luis Lavoura and Walter Grimus as well as the unknown referees very much for valuable comments. D. M. was supported by a DARPA Young Faculty Award and an Australian Research Coun-

cil DECRA fellowship. C. R. and M. M. were supported partly by the Chilean research project FONDECYT, Project No. 1140781, and No. 1140568, respectively.

as well as the group *Física de Altas Energías* of the Universidad del Bío-Bío.

-
- [1] K. A. Olive *et al.* [Particle Data Group Collaboration], “Review of Particle Physics,” *Chin. Phys. C* **38**, 090001 (2014).
 - [2] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, “Global fit to three neutrino mixing: critical look at present precision,” *J. High Energy Phys.* **12**, 123 (2012) [arXiv:1209.3023 [hep-ph]].
 - [3] S. F. King and C. Luhn, “Neutrino Mass and Mixing with Discrete Symmetry,” *Rept. Prog. Phys.* **76**, 056201 (2013) [arXiv:1301.1340 [hep-ph]].
 - [4] E. Ma, “Neutrino mass matrix from $S(4)$ symmetry,” *Phys. Lett. B* **632**, 352 (2006) [arXiv:hep-ph/0508231].
 - [5] E. Ma, “Neutrino Tribimaximal Mixing from $A(4)$ Alone,” *Mod. Phys. Lett. A* **25**, 2215 (2010) [arXiv:0908.3165 [hep-ph]].
 - [6] R. Gonzalez Felipe, H. Serodio and J. P. Silva, “Neutrino masses and mixing in A_4 models with three Higgs doublets,” *Phys. Rev. D* **88**, 015015 (2013) [arXiv:1304.3468 [hep-ph]].
 - [7] E. Ma, “Neutrino Mixing and Geometric CP Violation with $\Delta(27)$ Symmetry,” *Phys. Lett. B* **723**, 161 (2013) [arXiv:1304.1603 [hep-ph]].
 - [8] W. Grimus, L. Lavoura, and D. Neubauer, “A light pseudoscalar in a model with lepton family symmetry $O(2)$,” *J. High Energy Phys.* **07**, 051 (2008) [arXiv:0805.1175 [hep-ph]].
 - [9] G. Altarelli and F. Feruglio, “Discrete Flavor Symmetries and Models of Neutrino Mixing,” *Rev. Mod. Phys.* **82**, 2701 (2010) [arXiv:1002.0211 [hep-ph]].
 - [10] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, “A precision constraint on multi-Higgs-doublet models,” *J. Phys. G* **35**, 075001 (2008) [arXiv:0711.4022 [hep-ph]].
 - [11] F. Nagel, “New aspects of gauge-boson couplings and the Higgs sector,” Ph.D.-thesis, Heidelberg University (2004).
 - [12] M. Maniatis, A. von Manteuffel, O. Nachtmann and F. Nagel, “Stability and symmetry breaking in the general two-Higgs-doublet model,” *Eur. Phys. J. C* **48**, 805 (2006) [arXiv:hep-ph/0605184].
 - [13] C. C. Nishi, “CP violation conditions in N -Higgs-doublet potentials,” *Phys. Rev. D* **74**, 036003 (2006) [Erratum **76**, 119901 (2007)] [arXiv:hep-ph/0605153].
 - [14] M. Maniatis and O. Nachtmann, “Stability and symmetry breaking in the general three-Higgs-doublet model,” *J. High Energy Phys.* **02**, 058 (2015) [arXiv:1408.6833 [hep-ph]].
 - [15] J. Verschelde, “Algorithm 795: PHCpack: A General Purpose Solver for Polynomial systems by Homotopy Continuation,” *ACM Trans. Math. Software*, **25**, 2 (1999).
 - [16] M. Maniatis and D. Mehta, “Minimizing Higgs Potentials via Numerical Polynomial Homotopy Continuation,” *Eur. Phys. J. Plus* **127**, 91 (2012) [arXiv:1203.0409 [hep-ph]].